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## ‘MODERN’ PHILLIPS CURVES AND THE IMPLICATIONS FOR THE STATISTICAL PROCESS OF INFLATION

Bill Russell

# **‘MODERN’ PHILLIPS CURVES AND THE IMPLICATIONS FOR THE STATISTICAL PROCESS OF INFLATION**

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## **ABSTRACT**

‘Modern’ theories of the Phillips curve imply that inflation is an integrated, or near integrated’ process. This paper explains this implication and why these ‘modern’ theories are logically inconsistent with what is commonly known about the statistical process of inflation.

Keywords: Phillips curve, inflation, stationary, integrated, macroeconomics  
JEL Classification: C10, C20, E31.

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## 1. INTRODUCTION

Russell (2011, 2013), and Russell and Chowdhury (2013) assert that ‘modern’ theories of the Phillips curve that incorporate the property that the sum of the dynamic inflation terms equal one imply the theory simultaneously predicts inflation data follows an integrated process. Or if the sum is nearly one then a near integrated processes. This paper first explains this assertion with particular reference to the New Keynesian Phillips curve before arguing why the ‘modern’ theories the Phillips curve are inconsistent with what we commonly believe about the statistical process of inflation.

## 2. ‘MODERN’ PHILLIPS CURVES

The ‘modern’ Phillips curve literature can be thought of in terms of restrictions to the reduced form of the hybrid Phillips curve:

$$\pi_t = \delta_f E_t(\pi_{t+1}) + \delta_b \pi_{t-1} + \delta_z x_t + \varepsilon_t \quad (1)$$

where inflation,  $\pi_t$ , depends on expected inflation,  $E_t(\pi_{t+1})$ , conditioned on information available at time  $t$ , lagged inflation,  $\pi_{t-1}$ , a ‘forcing’ variable,  $x_t$ , and an error term,  $\varepsilon_t$ , due to the random errors of agents and the shocks to inflation. Inflation is defined as the first difference of the logarithm of the price level such that:  $\pi_t = p_t - p_{t-1}$  and lower case variables are in natural logarithms. There are numerous measures of the ‘forcing’ variable in the Phillips curve literature including the gap between the unemployment rate and its long-run level, the gap between real and potential output, real marginal costs, labour’s income share and the markup of price on unit labour costs.

In the purely backward looking adaptive expectations Phillips curve model of Friedman (1968) and Phelps (1967)  $\delta_f = 0$  and  $\delta_b = 1$ . In contrast, the New Keynesian (NK) Phillips Curve models of Clarida, Galí and Gertler (1999) and Svensson (2000) agents employ rational expectations and are purely forward-looking resulting in  $\delta_f = 1-d$  and  $\delta_b = 0$  where  $d$  is the rate of time discount. Finally, the hybrid models of Galí and Gertler (1999) and Galí *et al.* (2001) incorporate agents that are both backward and forward looking and  $\delta_f + \delta_b = 1-d$ .

The discount rate,  $d$ , is not identified explicitly in the theories without supplementary assumptions. However, assuming risk neutral agents, a symmetric loss function around the profit maximising price and an annual real interest rate of around 4 per cent then  $d$  is approximately 0.04 and 0.01 on an annual and quarterly basis respectively. Most empirical work on the NK and hybrid models proceed on the assumption that  $d = 0$ . Consequently, at an empirical level all three models predict that  $\delta_f + \delta_b = 1$  in equation (1) and the standard interpretation of this prediction is that the long-run Phillips curve is ‘vertical’. Finally, note that Russell and Chowdhury (2013) propose the statistical process consistent (SPC) Phillips curve which is also nested in equation (1) with  $\delta_f = 0$  and  $0 \leq \delta_b < 1$ .

### 3. A THEORETICAL SOLUTION

Consider one standard solution for the statistical process of inflation based on the New Keynesian Philips curve version of equation (1) which can be written:

$$\pi_t = \alpha E_t(\pi_{t+1}) + \beta x_t + v_t \quad (2)$$

where  $v_t$  is a series of ‘shocks’ to the inflation process. If  $\alpha < 1$  as in the New Keynesian model then the standard solution for equation (2) is:

$$\pi_t = E_t \sum_{j=1}^{\infty} \alpha^j (\beta x_{t+j} + v_{t+j}) \quad (3)$$

To solve equation (3) requires us to specify the statistical process of both the shocks,  $v_t$ , and the forcing variable,  $x_t$ . Note that there is no solution to equation (3) if  $\alpha = 1$  because the future is not discounted.

Ignoring the forcing variable for simplicity and, as we have assumed  $\alpha < 1$ :

$$\pi_t = E_t \sum_{j=1}^{\infty} \alpha^j v_{t+j} \quad (4)$$

If we also assume the shocks are a serially correlated autoregressive process

$$v_t = \rho v_{t-1} + \varepsilon_t \quad (5)$$

where  $\varepsilon_t$  is a white noise process then the solution to equation (4) is:

$$\pi_t = \frac{1}{1-\rho} v_t \quad (6)$$

implying inflation is a white noise stationary process. This theoretical approach leads to at least one solution that contradicts the assertion that inflation is an integrated, or near integrated, process in a ‘modern’ theory of the Phillips curve. Furthermore, this approach suggests that inflation can follow any statistical process depending on the specification of the statistical process of the shocks to inflation. We return to this issue in Section 5.

#### 4. AN EMPIRICAL SOLUTION

Consider the following. Estimate a structural NK Phillips curve with ordinary least squares:

$$\pi_t = \delta_1 \pi_{t+1} + \delta_2 x_t + \epsilon_t \quad (7)$$

where the error terms,  $\epsilon_t$ , are white noise and the ‘forcing variable’,  $x_t$ , is a stationary process.

Assume there are two mutually exclusive and exhaustive states of the world where inflation,  $\pi_t$ , is either an integrated I(1) or stationary I(0) process. Consider the first state when inflation is I(1). As the forcing variable  $x_t$  is I(0) it will not enter the asymptotics of the estimation and can be ignored. In this case, standard cointegration theory implies that  $\delta_1 = 1$ . If this is not the case then  $\Delta\pi_t$  would be I(1) implying that  $\pi_t$  is I(2) which contradicts the initial assumption that  $\pi_t$  is I(1). Alternatively, in the second state when inflation is I(0) then  $\delta_1 < 1$ . Again if this is not the case and  $\delta_1 = 1$  then this would imply that inflation is I(1) which contradicts our initial assumption.

The logic of estimating equation (7) suggests that (i) if inflation is I(1) then  $\delta_1 = 1$  and (ii) if inflation is I(0) then  $\delta_1 < 1$ . The converse is equally true. If  $\delta_1 = 1$  in the theory then inflation needs to be I(1) and if  $\delta_1 < 1$  then inflation needs to be I(0) so that the theory is ‘true’ in the sense of being consistent with the data. By implication if  $\delta_1$  is very close but not equal to 1 in the theory then inflation needs to be a near integrated process so that the theory is ‘true’.

#### 5. CONCLUSION

The theoretical New Keynesian solution in Section 3 suggests that any statistical process is possible for inflation depending on the process assumed for the shocks to inflation. However, the empirical solution in Section 4 suggests that if the New Keynesian model is ‘scientific’ in the sense of Popper (1959) and a valid description of the data then inflation must be an integrated process so that the estimated sum of the dynamic inflation terms is 1. Therefore, the

range of valid statistical process for the shocks in the theoretical solution in Section 3 is limited to only integrated processes if we believe the theory to be ‘valid’.

Turn now to what we know about the statistical process of inflation. First, inflation in the developed economies over the past 60 years appears to be bounded below around zero and above at some moderate rate. The ‘bounded’ nature of inflation implies it cannot be an integrated process. Second, if inflation is an  $I(1)$  process then the price level is an  $I(2)$  process. Given prices are conceptually non-negative then this means the price level has a lower boundary of zero and so the price level cannot be  $I(2)$  and, in turn, this rules out inflation as an integrated  $I(1)$  process.

What then is the statistical process of inflation? The rhetoric underpinning ‘modern’ Phillips curve theories of inflation argue the long-run rate of inflation depends on the setting of monetary policy and inflation will vary around that long-run rate. A change in monetary policy will see inflation converge on, and vary around, a new long-run rate. Therefore, we would expect inflation to be a stationary process around a shifting mean. And that the shifts are due to changes in monetary policy which are discrete and possibly frequent.

Therefore the analysis above leads us to be equally confident about the three following propositions. (i) Inflation is stationary and not an integrated process. (ii) The sum of the estimated dynamic inflation terms must therefore be less than 1. And (iii) ‘modern’ Phillip curve theories where the dynamic inflation terms sum to one are inconsistent with the statistical process of inflation and are not valid descriptions of the inflation.

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